Part 2.4: Inverse Normal

The next thing we need to do is the reverse of what we have done so far… when we are given a probability we need to find the number that it is up to, more than, or between. Let's look at an example.

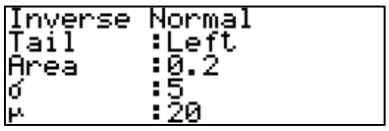
Example

The time spent by a teacher at work in the holidays is normally distributed with a mean of 20 hours and a standard deviation of 5 hours.

a. What amount of time do the bottom 20% of teachers work?
b. What amount of time do the top 20% of teachers work?
c. What amount of time do the middle 40% of teachers work?

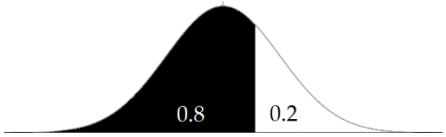
Answer (Graphics Calculator)

a. This time we go to InvN (Inverse Normal) and the tail is to the left (this is the default on the old calculators). The area is 0.2, the mean is 20 and the standard deviation is 5.



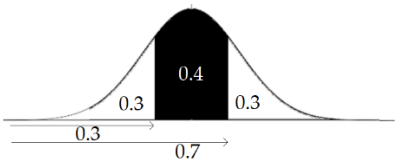
This gives us an answer of 15.8 hours, so in context, the bottom 20% of teachers work less than 15.8 hours.

b. Again we have an inverse normal… if we look at the graph we either want the top 0.2 or the bottom 0.8.



If you have an old calculator you need to put in 0.8, otherwise you can say it is a right tail of 0.2. This gives us an answer of 24.2 which in context means that the top 20% of teachers work more than 24.2 hours in the holidays.

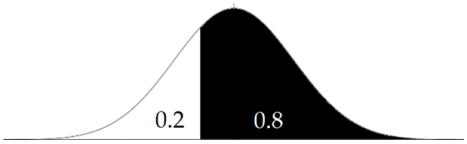
c. On the new calculators this is really easy… you just change the tail to central, otherwise if you draw a diagram you see what is going on and work out the area from the left.



This gives us (17.4, 22.6) which in context means the middle 40% of teachers work between 17.4 hours and 22.6 hours.

Answer (Tables)

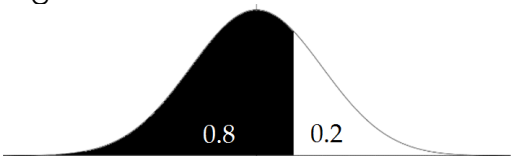
a. The first thing we need to do is draw a diagram.



Now, what we need is the area from the middle to the point that we want. In this case the area is 0.3. If we look up 0.3 in the body of the table we can see that the Z score that relates to it is 0.841 as 0.2996 + 3 is the closest we can get to 0.3… now obviously the point that we want is below the mean so the Z score is -0.841. If we put this into our formula we get $-0.841 = \frac{X-20}{5}$

If we solve this we find out that X = 15.8, which in context means the bottom 20% of teachers work less than 15.8 hours in the holidays.

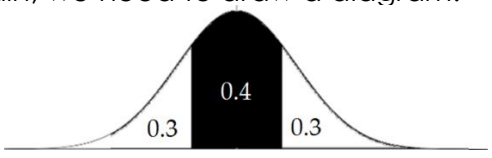
b. Again, the first thing we need to do is draw a diagram.



Again, the area between the mean and the point we want is 0.3, giving us a Z score of 0.841, and because this value is more than the mean, we leave it positive this time. If we put this into our formula we get $0.841 = \frac{X-20}{5}$

If we solve this we find out that X = 24.2, which in context means the top 20% of teachers work more than 24.2 hours in the holidays.

c. Again, we need to draw a diagram.



With this we can see that the area between the mean and the point that we want is 0.2 in each direction. If we look up 0.2 in the tables we find that the Z score that it lines up with is 0.524 (0.1985+14 is as close to 0.2 as we can get).

We need to solve this again using the same method as before, but this time with **both** the positive and negative numbers.
$0.524 = \frac{X-20}{5}$ and $-0.524 = \frac{X-20}{5}$

Which gives us 22.6 and 17.4 which in context means the middle 40% of teachers work between 17.4 hours and 22.6 hours in the holidays.

Exercise 2.4

1. The time spent in freefall in when skydiving is normally distributed with a mean of 20 seconds and a standard deviation of 2 seconds. What times do the middle 50% of jumpers freefall for?
2. The width of an A4 piece of paper is normally distributed with a mean of 210 mm and a standard deviation of 0.2 mm. Over what size are the top 20% of A4 pieces of paper?
3. The length of a football field is normally distributed with a mean of 100 m and a standard deviation of 3 m. Under what size are the smallest 10% of football fields?
4. The time taken to boil a kettle is normally distributed with a mean of 94 seconds and a standard deviation of 6 seconds. What times do the middle 40% of boils fall between?
5. The temperature in a glass house is normally distributed with a mean of 30 °C and a standard deviation of 3 °C. Under what temperature are the coldest 5% of days?
6. The weight of a block of butter is normally distributed with a mean of 510 g and a standard deviation of 4 g. Over what weight are the top 4% of blocks of butter?
7. The height of rugby players is normally distributed with a mean of 1.86 m and a standard deviation of 0.08 m. Between what weights are the middle 80% of rugby players?
8. The length of an elephant's trunk is normally distributed with a mean of 175 cm and a standard deviation of 10 cm. What length are the shortest 10% of elephant trunks under?