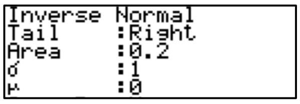
Part 2.5: Finding the Mean and Standard Deviation

For Merit and Excellence we are sometimes required to find the mean and or the standard deviation based on other pieces of information that is given to us. To do this we need to use the formula that is given at the top of the formula sheet: $Z = \frac{X-\mu}{\sigma}$. Sometimes this will require using simultaneous equations, and sometimes it will just involve a straight use of the formula. Let's look at some examples…

Example 1 – Finding the standard deviation
The amount of time that it takes to change a nappy is normally distributed with a mean of 40 seconds. The worst 20% of nappies take more than 60 seconds to change. Calculate the standard deviation for the time it takes to change a nappy.

Answer (Graphics Calculator)
The first thing we need to do is calculate the Z-score. To do this we are using a 'standard normal' curve… this means for the moment the mean is zero and the standard deviation is one, and we know the right tail is 0.2. We then get:



This gives us a Z score of 0.8416.
We then use the formula $Z = \frac{X-\mu}{\sigma}$ and substitute in the values… $0.8416 = \frac{60-40}{\sigma}$ and solve it to find that σ = 23.8 minutes.

Answer (Tables)
The first thing we need to do is work out the Z-score using the tables just like we did in the last section. The difference between the mean and the point we are after is 0.3 (draw a diagram to help) which when we look it up in the tables we get a Z-score of 0.841. We then use the formula $Z = \frac{X-\mu}{\sigma}$ and substitute in the values… $0.841 = \frac{60-40}{\sigma}$ and solve it to find that σ = 23.8 minutes.

Example 2 – Finding the mean
The marks in a test are normally distributed with a standard deviation of 20%. If the top 10% of students score more than 90%, what is the mean test mark?

Answer (Graphics Calculator)
Again, the first thing we need to do is calculate the Z-score, we do this using the same process as before… Tail: right, Area: 0.1, σ: 1, μ: 0.
This gives us a Z-score of 1.282.
We then substitute in the values into our formula… $1.282 = \frac{90-\mu}{20}$ and find that μ = 64.4%.

Answer (Tables)
We again use the same process, finding a z-score of 1.282. We put this into the formula $Z = \frac{X-\mu}{\sigma}$ and substitute in the values… $1.282 = \frac{90-\mu}{20}$ and solve it to find that μ = 64.4%.

Example 3 – Finding both the mean and the standard deviation.
The heights of Great Spotted Kiwi Birds are normally distributed. The shortest 10% are under 44 cm tall and the tallest 20% are more than 48 cm tall. Calculate the mean and standard deviation of the kiwi's heights.

Answer
In this case because we need to find both the mean and standard deviation, we need to use simultaneous equations. We find the Z score for each part and the form the following equations.

$$-1.281 = \frac{44-\mu}{\sigma} \text{ and } 0.842 = \frac{48-\mu}{\sigma}$$

If we rearrange these to get

$$-1.281\sigma = 44 - \mu \text{ and } 0.842\sigma = 48 - \mu$$

We can then equate them to:

$$-1.281\sigma - 44 = 0.842\sigma - 48$$

Which we can solve to find σ = 1.884 and then substitute this in to find μ = 46.4.

Exercise 2.5

1. The time an office worker spends at work is normally distributed with a mean of 8 hours. On the busiest 10% of days he spends more than 9 hours at work. Calculate the standard deviation.
2. The time a marathon runner takes to complete a marathon is normally distributed with a mean of 4 hours. The fastest 10% of runners finish in under 2.5 hours. Calculate the standard deviation for the time to complete a marathon.
3. The weight of sugar in a bag is normally distributed with a standard deviation of 4g. If the lightest 5% of bags are less than 500 g, what is the mean weight of sugar in the bag?
4. The time spent watching television in a week is normally distributed with a standard deviation of 2 hours. If the top 5% of people spend more than 10 hours watching TV a week, what is the mean time spent watching TV a week?
5. The lengths of boats in a marina are normally distributed with a mean of 5 m. If the shortest 10% of boats are less than 2 m what is the standard deviation for the length of the boats?
6. The length of a foot long sub is normally distributed. If the shortest 10% of subs are less than 30 cm and the longest 20% are more than 31 cm, what is the mean and standard deviation for the lengths of foot-long subs?
7. The height of 5 year olds are normally distributed. The shortest 20% of 5 year olds are under 100 cm and the tallest 30% are more than 130cm. Calculate the mean and standard deviation for the height of five year olds.
8. The weight of an unladen swallow is normally distributed. The lightest 30% are less than 13.5 g and the heaviest 40% are more than 14.5g. Calculate the mean and standard deviation for the weight of unladen swallows.